Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topics: 3D Geometry

***Do your work with your whole heart, and you will succeed - there's so little competition.*

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. A mirror and a source of light are situated at the origin O and a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the DRs of the normal to the plane of mirror are 1, – 1, 1, then DCs for the reflected ray are
(a) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (b) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$
(c) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

2. The direction cosines of a line satisfy the relations $\lambda(l+m)=n$ and $mn+nl+lm=0$. The value of λ, for which the two lines are perpendicular to each other, is
(a) 1 (b) 2
(c) 1/2 (d) none of these

3. Let P be any point on the plane $lx+my+nz=p$ and Q be a point on the line OP such that $OP \cdot OQ = p^2$. The locus of the point Q is
(a) $lx+my+nz-p=x^2+y^2+z^2$
(b) $lx+my+nz=p(x^2+y^2+z^2)$
(c) $p(lx+my+nz)=x^2+y^2+z^2$
(d) $x^2+y^2+z^2=p^2$

4. The coordinates of a point on the line $\frac{x-1}{2}=\frac{y+1}{-3}=z$ at a distance $4\sqrt{14}$ from the point (1, – 1, 0) nearer the origin are
(a) (9, – 13, 4) (b) $(8\sqrt{14}, -12, -1)$
(c) $(-8\sqrt{14}, 12, 1)$ (d) (–7, 11, – 4)

5. The equation of motion of a point in space is $x=2t$, $y=-4t$, $z=4t$, where it measured in hour and the coordinates of moving point in kilometers. The distance of the point from the starting point O(0, 0, 0) in 10 hours is
(a) 20 km (b) 40 km
(c) 60 km (d) 55 km

6. The locus of the point, the sum of squares of whose distances, from the planes $x-z=0$, $x-2y+z=0$ and $x+y+z=0$ is 36 is
(a) $x^2+y^2+z^2=6$ (b) $x^2+y^2+z^2=36$
(c) $x^2+y^2+z^2=216$ (d) $x^{-2}+y^{-2}+z^{-2}=\frac{1}{36}$

7. The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy=c^2, z=0$ if c is equal to
(a) ± 1 (b) $\pm\frac{1}{3}$
(c) $\pm\sqrt{5}$ (d) none of these

8. The four lines drawn from the vertices of any tetrahedron to the centroid of the opposite faces meet in a point whose distance from each vertex is k times the distance from each vertex to the opposite face, where k is
(a) 1/3 (b) 1/2
(c) 3/4 (d) 5/4

9. Which of the statement is true ? The coordinate planes divide the line joining the points (4, 7, – 2) and (–5, 8, 3)
(a) all externally
(b) two externally and one internally
(c) two internally and one externally
(d) none of the above

10. The pair of lines whose direction cosines are given by the equations $3l+m+5n=0$, $6mn-2nl+5lm=0$, are
(a) parallel
(b) perpendicular
(c) inclined at $\cos^{-1}\left(\frac{1}{6}\right)$
(d) none of the above

11. The distance of the point $A(-2, 3, 1)$ from the line PQ through $P(-3, 5, 2)$ which make equal angles with the axes is
(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{\frac{14}{3}}$
(c) $\frac{16}{\sqrt{3}}$ (d) $\frac{5}{\sqrt{3}}$

12. The equation of the plane through the point $(2, 5, -3)$ perpendicular to the planes $x + 2y + 2z = 1$ and $x - 2y + 3z = 4$ is

(a) $3x - 4y + 2z - 20 = 0$
(b) $7x - y + 5z = 30$
(c) $x - 2y + z = 11$
(d) $10x - y - 4z = 27$

13. The equation of the plane through the point $(0, -4, -6)$ and $(-2, 9, 3)$ and perpendicular to the plane $x - 4y - 2z = 8$ is

(a) $3x + 3y - 2z = 0$ (b) $x - 2y + z = 2$
(c) $2x + y - z = 2$ (d) $5x - 3y + 2z = 0$

14. The equation of the plane passing through the points $(3, 2, -1)$, $(3, 4, 2)$ and $(7, 0, 6)$ is $5x + 3y - 2z = \lambda$, where λ is

(a) 23 (b) 21
(c) 19 (d) 27

15. A variable plane which remains at a constant distance p from the origin cuts the coordinate axes in A, B, C. The locus of the centroid of the tetrahedron $OABC$ is $y^2z^2 + z^2x^2 + x^2y^2 = kx^2y^2z^2$, where k is equal to

(a) $9p^2$ (b) $\frac{9}{p^2}$
(c) $\frac{7}{p^2}$ (d) $\frac{16}{p^2}$

16. The line joining the points $(1, 1, 2)$ and $(3, -2, 1)$ meets the plane $3x + 2y + z = 6$ at the point

(a) $(1, 1, 2)$ (b) $(3, -2, 1)$
(c) $(2, -3, 1)$ (d) $(3, 2, 1)$

17. The point on the line $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+5}{-2}$ at a distance of 6 from the point $(2, -3, -5)$ is

(a) $(3, -5, -3)$ (b) $(4, -7, -9)$
(c) $(0, 2, -1)$ (d) $(-3, 5, 3)$

18. The plane passing through the point $(5, 1, 2)$ perpendicular to the line $2(x - 2) = y - 4 = z - 5$ will meet the line in the point

(a) $(1, 2, 3)$ (b) $(2, 3, 1)$
(c) $(1, 3, 2)$ (d) $(3, 2, 1)$

19. The point equidistant from the points $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ and $(0, 0, 0)$ is

(a) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (b) (a, b, c)
(c) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ (d) none of these

20. P, Q, R, S are four coplanar points on the sides AB, BC, CD, DA of a skew quadrilateral. The product $\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RD} \cdot \frac{DS}{SA}$ equals

(a) -2 (b) -1
(c) 2 (d) 1

21. The angle between any two diagonals of a cube is

(a) $\cos\theta = \frac{\sqrt{3}}{2}$ (b) $\cos\theta = \frac{1}{\sqrt{2}}$
(c) $\cos\theta = \frac{1}{3}$ (d) $\cos\theta = \frac{1}{\sqrt{6}}$

22. The acute angle between two lines whose direction cosines are given by the relation between $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$ is

(a) $\pi / 2$ (b) $\pi / 3$
(c) $\pi / 4$ (d) none of these

23. The lines $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{-1}$, $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{4}$ are

(a) parallel lines
(b) intersecting lines
(c) perpendicular skew lines
(d) none of the above

24. The direction cosines of the line drawn from $P(-5, 3, 1)$ to $Q(1, 5, -2)$ is

(a) $(6, 2, -3)$ (b) $(2, -4, 1)$
(c) $(-4, 8, -1)$ (d) $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

25. The coordinates of the centroid of triangle ABC where A, B, C are the points of intersection of the plane $6x + 3y - 2z = 18$ with the coordinate axes are

(a) $(1, 2, -3)$ (b) $(-1, 2, 3)$
(c) $(-1, -2, -3)$ (d) $(1, -2, 3)$

26. The intercepts made on the axes by the plane which bisects the line joining the points $(1, 2, 3)$ and $(-3, 4, 5)$ at right angles are

(a) $\left(-\frac{9}{2}, 9, 9\right)$ (b) $\left(\frac{9}{2}, 9, 9\right)$
(c) $\left(9, -\frac{9}{2}, 9\right)$ (d) $\left(9, \frac{9}{2}, 9\right)$

27. A lines makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ is

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$
(c) 3 (d) none of these

28. A variable plane passes through the fixed point (a, b, c) and meets the axes at A, B, C. The locus of the point of intersection of the planes through A, B, C and parallel to the coordinate planes is

(a) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$ (b) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$
(c) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = -2$ (d) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = -1$

29. A plane moves such that its distance from the origin is a constant p. If it intersects the coordinate axes at A, B, C then the locus of the centroid of the triangle ABC is

(a) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$ (b) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{9}{p^2}$
(c) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{2}{p^2}$ (d) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{4}{p^2}$

30. The distance between two points P and Q is d and the length of their projections of PQ on the coordinate planes are d_1, d_2, d_3. Then $d_1^2 + d_2^2 + d_3^2 = kd^2$, where k is
(a) 1 (b) 5
(c) 3 (d) 2

31. The line $\frac{x}{2} = -\frac{y}{3} = \frac{z}{1}$ is vertical. The direction cosines of the line of greatest slope in the plane $3x - 2y + z = 5$ are proportional to
(a) $(16, 11, -1)$ (b) $(-11, 16, 1)$
(c) $(16, 11, 1)$ (d) $(11, 16, -1)$

32. The symmetric form of the equations of the line $x + y - z = 1, 2x - 3y + z = 2$ is
(a) $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ (b) $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{5}$
(c) $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{5}$ (d) $\frac{x}{3} = \frac{y}{2} = \frac{z}{5}$

33. The equation of the plane which passes through the x-axis and perpendicular to the line $\frac{(x-1)}{\cos\theta} = \frac{(y+2)}{\sin\theta} = \frac{(z-3)}{0}$ is
(a) $x\tan\theta + y\sec\theta = 0$ (b) $x\sec\theta + y\tan\theta = 0$
(c) $x\cos\theta + y\sin\theta = 0$ (d) $x\sin\theta - y\cos\theta = 0$

34. The edge of a cube is of length of a. The shortest distance between the diagonal of a cube and an edge skew to it is
(a) $a\sqrt{2}$ (b) a
(c) $\frac{\sqrt{2}}{a}$ (d) $\frac{a}{\sqrt{2}}$

35. The equation of the plane passing through the intersection of the planes $2x - 5y + z = 3$ and $x + y + 4z =$`5 and parallel to the plane $x + 3y + 6z = 1$ is $x + 3y + 6z = k$, where k is
(a) 5 (b) 3
(c) 7 (d) 2

36. The lines which intersect the skew lines $y = mx, z = c$; $y = -mx, z = -c$ and the x-axis lie on the surface
(a) $cz = mxy$ (b) $cy = mxz$
(c) $xy = cmz$ (d) none of these

37. The equation of the line passing though the point $(1, 1, -1)$ and perpendicular to the plane $x - 2y - 3z = 7$ is
(a) $\frac{x-1}{-1} = \frac{y-1}{2} = \frac{z+1}{3}$ (b) $\frac{x-1}{-1} = \frac{y-1}{-2} = \frac{z+1}{3}$
(c) $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z+1}{-3}$ (d) none of these

38. The plane $4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $5x + 3y + 10z = 25$. The equation of the plane in its new position is $x - 4y + 6z = k$, where k is
(a) 106 (b) -89
(c) 73 (d) 37

39. A plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (a, a, a). Then the equation of the plane is $x + y + z = p$, where p is
(a) a (b) $3/a$
(c) $a/3$ (d) $3a$

40. If from the point $P(a, b, c)$ perpendiculars PL, PM be drawn to YOZ and ZOX planes, then the equation of the plane OLM is
(a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ (b) $\frac{x}{a} - \frac{y}{b} + \frac{z}{c} = 0$
(c) $\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0$ (d) $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$

41. A variable plane makes with the coordinate planes, a tetrahedron of constant volume $64k^3$. Then the locus of the centroid of tetrahedron is the surface
(a) $xyz = 6k^3$ (b) $xy + yz + zx = 6k^2$
(c) $x^2 + y^2 + z^2 = 8k^2$ (d) none of these

42. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = k$, meets the coordinate axes at A, B, C such that the centroid of the triangle ABC is the point (a, b, c). Then k is
(a) 3 (b) 2
(c) 1 (d) 5

43. The perpendicular distance of the origin from the plane which makes intercepts 12, 3 and 4 on x, y, z axes respectively, is
(a) 13 (b) 11
(c) 17 (d) none of these

44. A plane meets the coordinate axes at A, B, C and the foot of the perpendicular from the origin O to the plane is $P, OA = a, OB = b, OC = c$. If P is the centroid of the triangle ABC, then
(a) $a + b + c = 0$ (b) $|a| = |b| = |c|$
(c) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$ (d) none of these

45. $ABCD$ is a tetrahedron. A_1, B_1, C_1, D_1 are respectively the centroids of the triangles BCD, ACD, ABD and ABC; AA_1, BB_1, CC_1, DD_1 divide one another in the ratio
(a) 1 : 1 (b) 2 : 1
(c) 3 : 1 (d) 1 : 3

46. A plane makes intercepts OA, OB, OC whose measurements are a, b, c on the axes OX, OY, OZ. The area of the triangle ABC is
(a) $\frac{1}{2}(ab + bc + ca)$
(b) $\frac{1}{2}(a^2b^2 + b^2c^2 + c^2a^2)^{1/2}$
(c) $\frac{1}{2}abc\,(a + b + c)$
(d) $\frac{1}{2}(a + b + c)^2$

47. The projections of a line on the axes are 9, 12 and 8. The length of the line is
(a) 7 (b) 17
(c) 21 (d) 25

48. If P, Q, R, S are the points $(4, 5, 3)$ $(6, 3, 4)$, $(2, 4, -1)$, $(0, 5, 1)$, the length of projection RS on PQ is

(a) 4/3 (b) 2/3
(c) 4 (d) 6

49. The distance of the point $P(-2, 3, 1)$ from the line QR, through $Q(-3, 6, 2)$ which makes equal angles with the axes is
(a) 3 (b) 8
(c) $\sqrt{2}$ (d) $2\sqrt{2}$

50. The points $(8, -5, 6)$, $(11, 1, 8)$, $(9, 4, 2)$ and $(6, -2, 0)$ are the vertices of a
(a) rhombus (b) square
(c) rectangle (d) parallelogram

51. The straight lines whose direction cosines are given by $al + bm + cn = 0$, $fmn + gnl + hlm = 0$ are perpendicular if
(a) $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$
(b) $\frac{a^2}{f} + \frac{b^2}{g} + \frac{c^2}{h} = 0$
(c) $a^2(g + h) + b^2(h + f) + c^2(f + g) = 0$
(d) none of the above

52. The three planes $4y + 6z = 5$; $2x + 3y + 5z = 5$; $6x + 5y + 9z = 10$.
(a) meet in a point (b) have a line in common
(c) form a triangular prism (d) none of these

53. The line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4}$ meets the plane $x + 2y + 3z = 14$, in the point
(a) $(3, -2, 5)$ (b) $(3, 2, -5)$
(c) $(2, 0, 4)$ (d) $(1, 2, 3)$

54. The foot of the perpendicular from $P(1, 0, 2)$ to the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ is the point
(a) $(1, 2, -3)$ (b) $\left(\frac{1}{2}, 1, -\frac{3}{2}\right)$
(c) $(2, 4, -6)$ (d) $(2, 3, 6)$

55. The length of the perpendicular from $(1, 0, 2)$ on the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ is
(a) $\frac{3\sqrt{6}}{2}$ (b) $\frac{6\sqrt{3}}{5}$
(c) $3\sqrt{2}$ (d) $2\sqrt{3}$

56. The plane containing the two lines $\frac{x-3}{1} = \frac{y-2}{4} = \frac{z-1}{5}$ and $\frac{x-2}{1} = \frac{y+3}{-4} = \frac{z+1}{5}$ is $11x + my + nz = 28$, where
(a) $m = -1, n = 3$ (b) $m = 1, n = -3$
(c) $m = -1, n = -3$ (d) $m = 1, n = 3$

57. The projection of the line $\frac{x+1}{-1} = \frac{y}{2} = \frac{z-1}{3}$ on the plane $x - 2y + z = 6$ is the line of intersection of this plane with the plane
(a) $2x + y + 2 = 0$ (b) $3x + y - z = 2$
(c) $2x - 3y + 8z = 3$ (d) none of these

58. A variable plane passes through a fixed point $(1, -2, 3)$ and meets the coordinate axes in A, B, C. The locus of the point of intersection of the planes through A, B, C parallel to the coordinate planes is the surface
(a) $xy - \frac{1}{2}yz + \frac{1}{3}zx = 6$ (b) $yz - 2zx + 3xy = xyz$
(c) $xy - 2yz + 3zx = 3xyz$ (d) none of these

59. The distance of the point $(2, 1, -2)$ from the line $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-3}$ measured parallel to the plane $x + 2y + z = 4$ is
(a) $\sqrt{10}$ (b) $\sqrt{20}$
(c) $\sqrt{5}$ (d) $\sqrt{30}$

60. The shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is
(a) $2\sqrt{3}$ (b) $4\sqrt{3}$
(c) $3\sqrt{6}$ (d) $5\sqrt{6}$

61. The area of the triangle whose vertices are at the points $(2, 1, 1)$, $(3, 1, 2)$, $(-4, 0, 1)$ is
(a) $\sqrt{19}$ (b) $\frac{1}{2}\sqrt{19}$
(c) $\frac{1}{2}\sqrt{38}$ (d) $\frac{1}{2}\sqrt{57}$

62. The equation to the plane through the points $(2, -1, 0)$, $(3, -4, 5)$ parallel to a line with direction cosines proportional to 2, 3, 4 is $9x - 2y - 3z = k$, where k is
(a) 20 (b) −20
(c) 10 (d) −10

63. Through a point $P(f, g, h)$ a plane is drawn at right angles to OP, to meet the axes in A, B, C. If $OP = r$, the centroid of the triangle ABC is
(a) $\left(\frac{f}{3r}, \frac{g}{3r}, \frac{h}{3r}\right)$ (b) $\left(\frac{r^2}{3f^2}, \frac{r^2}{3g^2}, \frac{r^2}{3h^2}\right)$
(c) $\left(\frac{r^2}{3f}, \frac{r^2}{3g}, \frac{r^2}{3h}\right)$ (d) none of these

64. The plane $lx + my = 0$ is rotated about its line of intersection with the xOy plane through an angle α. Then the equation of the plane is $lx + my + nz = 0$, where n is
(a) $\pm\sqrt{(l^2 + m^2)}\cos\alpha$ (b) $\pm\sqrt{(l^2 + m^2)}\sin\alpha$
(c) $\pm\sqrt{(l^2 + m^2)}\tan\alpha$ (d) none of these

65. The condition for the lines $x = az + b$, $y = cz + d$ and $x = a_1z + b_1$, $y = c_1z + d_1$ to be perpendicular is
(a) $ac_1 + a_1c = 1$ (b) $aa_1 + cc_1 + 1 = 0$
(c) $bc_1 + b_1c + 1 = 0$ (d) none of these

Answers

Objective Questions Type I [Only one correct answer]

1.	(d)	2.	(b)	3.	(c)	4.	(d)	5.	(c)	6.	(b)	7.	(c)	8.	(c)	9.	(c)	10.	(c)
11.	(b)	12.	(d)	13.	(c)	14.	(a)	15.	(d)	16.	(b)	17.	(b)	18.	(a)	19.	(c)	20.	(d)
21.	(c)	22.	(b)	23.	(c)	24.	(d)	25.	(a)	26.	(a)	27.	(a)	28.	(b)	29.	(b)	30.	(d)
31.	(d)	32.	(c)	33.	(c)	34.	(d)	35.	(c)	36.	(b)	37.	(c)	38.	(a)	39.	(d)	40.	(c)
41.	(a)	42.	(a)	43.	(d)	44.	(b)	45.	(c)	46.	(b)	47.	(b)	48.	(a)	49.	(d)	50.	(b)
51.	(a)	52.	(b)	53.	(d)	54.	(b)	55.	(a)	56.	(c)	57.	(a)	58.	(b)	59.	(d)	60.	(b)
61.	(c)	62.	(a)	63.	(c)	64.	(c)	65.	(b)										